J Stat Phys
DOI 10.1007/s10955-012-0678-4

The Effects of Sacred Value Networks Within an Evolutionary, Adversarial Game

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Received: 24 August 2012 / Accepted: 14 December 2012
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Abstract The effects of personal relationships and shared ideologies on levels of crime and the formation of criminal coalitions are studied within the context of an adversarial, evolutionary game first introduced in Short et al. (Phys. Rev. E 82:066114, 2010). Here, we interpret these relationships as connections on a graph of N players. These connections are then used in a variety of ways to define each player's "sacred value network"—groups of individuals that are subject to special consideration or treatment by that player. We explore the effects on the dynamics of the system that these networks introduce, through various forms of protection from both victimization and punishment. Under local protection, these networks introduce a new fixed point within the game dynamics, which we find through a continuum approximation of the discrete game. Under more complicated, extended protection, we numerically observe the emergence of criminal coalitions, or "gangs". We also find that a high-crime steady state is much more frequent in the context of extended protection networks, in both the case of Erdős-Rényi and small world random graphs.

Keywords Crime dynamics · Sacred values · Evolutionary game theory

1 Introduction

The members of gangs and insurgent networks—which we shall simply refer to as "criminals"—are usually well known within their local communities, and are often sheltered from authorities by non-criminal community members. Criminals may be integrated with the local community because they are feared or loved; are part of large familial groups; or in some cases because shared goals, ideals, or ethnic/religious identity form an impetus for criminal illicit activities. Such shared goals or ideals, ties of friendship and fear, and

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Published online: 11 January 2013

family allegiances can all be considered "sacred values" that will affect the relationship between criminals and the community. In general, sacred values are treated by their holders as inviolable or non-negotiable [16]. Attempts to induce individuals or groups to act in contravention of sacred values typically will elicit moral outrage, or worse, violent counter-attacks. Sacred values thus may drive individual and group actions even where those actions are demonstrably against economic self-interest, or are guaranteed to lead to failure [2, 3]. Sacred values might also lead to inaction on the part of individuals or groups. For example, a taboo against "snitching" may be pervasive enough in a community to lead individuals to avoid reporting crime at all costs [1, 13].

The goal of this work is to incorporate the idea of sacred values into a mathematical framework previously used to model levels of criminal behavior within a society. We will consider modifications of the stochastic evolutionary game introduced in [14], which models the levels of both crime and cooperation with authorities in a society and is described in some detail below. Several variants of this model have already been characterized mathematically in [10, 11, 15] and tested with human experiments in [7]. This model predicts that a society will eventually reside in one of two equilibrium states: a high-crime, low cooperation state called "Dystopia"; and a low-crime, high cooperation state called "Utopia". We impose sacred values in this model via a static sacred values network, wherein each player is connected to a subset of other players in the game. The dynamics between two players will differ depending upon whether they are within each other's protected network or external to it.

As we will show, sacred values can stabilize a criminal population that would not be a possible steady state in the original model. The protection offered by sacred networks thus provides a reasonable setting to study the formation of gangs. The formation of network cliques is usually studied on static graphs where dynamic weighting functions dictate group membership; this approach is taken in [8, 9]. Our concept of a "gang" will rely on the underlying sacred values network, and will not include a dynamic weighting to determine group membership. Most of our results arise from numerical simulations on the stochastic game, though some of our results are found through a continuum equation that approximates our modified version of the stochastic game.

We begin by summarizing the game first described in [14]. Here, there are N players, and each player may adopt one of four possible "strategies": Paladin, Apathetic, Informant, or Villain. Paladins and Apathetics are not criminals, while Informants and Villains are. Paladins and Informants will report crimes and cooperate with authorities as witnesses, while Apathetics and Villains will not. The N person game proceeds in rounds. At the start of each round, one criminal is selected from the pool of Villains and Informants and one victim from the remaining $N-1$ players. Both players start the round with payoff 1. A crime then occurs wherein the criminal steals $\delta < 1$ from the victim's payoff and transfers it to himself. If the victim is an Apathetic or Villain, the authorities are not involved, so the criminal keeps the stolen δ. If the victim is a Paladin or Informant, however, then the authorities are alerted and a trial follows. Here, a pool of M potential witnesses is selected from the remaining $N-2$ players, and the perpetrator is convicted with a probability equal to the fraction of these witnesses who are Paladins or Informants. If the criminal is convicted, he returns the δ stolen to the victim and suffers an additional punishment loss $\theta < 1$. If the criminal is not convicted, he keeps the δ and the victim suffers an additional loss $\epsilon < 1-\delta$. This additional loss for failed reporting may correspond to a loss of faith in the system, or to retaliation by the community. After the criminal event is resolved, the active part of the game is over.

Each round thus described has a "winner" and a "loser", determined by their respective payoffs at the end of the criminal event; the loser is the player with the lower payoff. The

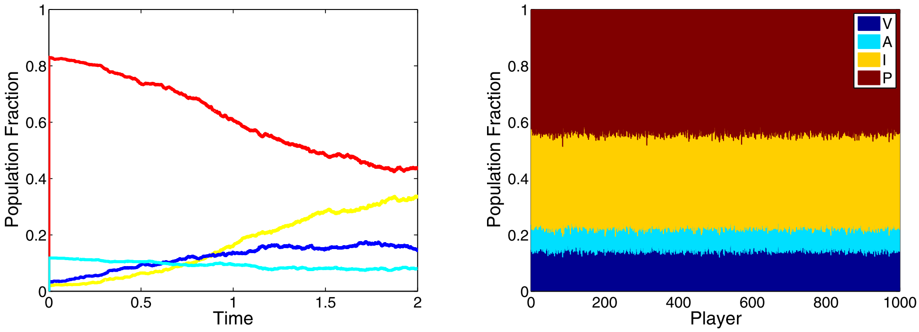

Fig. 1 Sample run of the game with a friendly local sacred values network: P is *red*, I is *yellow*, A is *cyan*, and V is *blue*. The parameters are $N = 1000$, $M = 5$, $\delta = 0.3$, $\theta = 0.6$, $\epsilon = 0.2$, and $p = 0.65$. The initial population fractions are $(P, I, V, A) = (0.8, 0.03, 0.03, 0.14)$. *On the left* is the population fraction by type as a function of time, and *on the right* is the population fraction in each players' LSVN at the final time step (Color figure online)

loser then has a chance to update his strategy. He does so by randomly choosing to "mimic" either himself or the winner, with probability proportional to the payoff of each. If the victimizer is mimicked, then the loser will copy that player's strategy exactly and finish the round as either an Informant or Villain. If the victim is mimicked, the loser always ends as a non-criminal type, and only adopts the winner's witnessing strategy. In this case, the loser will end the round as either a Paladin or an Apathetic. This asymmetry in mimicking is meant to reduce bias towards criminal strategies, as every round has a criminal type (the victimizer), but not necessarily a non-criminal type (the victim could be of a criminal type).

Note that there is no concept of personal relationships embedded in this game as describe in [14], and that players' strategy choices are determined solely by their own experiences, and in no way influenced by their connections to the other players. However, in the context of a sacred value network, a criminal is more likely to victimize a stranger than he is to rob his brother, and his brother is less likely to witness against him than a stranger is. By introducing a sacred values network for each player, which represents family, friends, or people with a shared ideology, we hope to model the formation of crime in an otherwise peaceful society and the development of organized crime or gangs. Recall that these notions are not unique to garden-variety criminals and gangs, but could also apply to insurgent and terrorist networks.

2 The Game with a Friendly Local Network

A simple model where sacred values are included can be defined as follows. Every player is connected to a subset of the N players, and these connections are always reciprocal. The collection of an individual player's connections is the player's local sacred values network (LSVN). That is, only the immediate neighbors of player k, with whom k has a direct tie, are in k's LSVN (see Fig. 4). A criminal Informant or Villain will not victimize anyone in his LSVN, but will rather choose his victims from its complement in N. Thus, player k will not victimize a friend, but will potentially victimize the friend of a friend. Additionally, if a witness to the crime is drawn from the criminal's LSVN, this witness will not convict the criminal regardless of the witness' type. For example, a Paladin in a criminal's sacred values network will behave like an Apathetic if he is drawn to witness against his friend.

Clearly the dynamics of this modified game will depend on the form of the sacred values connections. For simplicity, we will concentrate on Erdős-Rényi (ER) random graphs, in which each player's LSVN is produced by connecting to each other player with probability p, and the expected number of friendly connections per person is thus $p(N-1)$.

We now define for future reference the variables P, A, I, and V, which represent the fraction of the total population N that are Paladins, Apathetics, Informants, and Villains, respectively; hence, $P+A+I+V=1$. In Fig. 1, the outcome of a single realization of this game is displayed. After the game has progressed for some time, the composition by strategy type of each players' LSVN is roughly the same as everyone else's; this is seen in the right panel of the figure. As a result, a deterministic version of this game can be derived following the same approach as in [14]. Analyzing this deterministic system provides a great deal of information about how the stochastic system behaves. The deterministic system is given by four coupled ordinary differential equations (ODEs):

$$
\begin{aligned}
\dot{P} &= (1-p)(I+V)\bigg[(I+P)^2\frac{1-p}{2-\theta} + I\big[A+V+p(I+P)\big]\frac{1-\delta-\epsilon}{2-\epsilon} \\
&\qquad - P\big[A+V+p(I+P)\big]\frac{1+\delta}{2-\epsilon}\bigg], \\
\dot{I} &= (1-p)I\bigg[\big[A+V+p(I+P)\big]\bigg(P\frac{1+\delta}{2-\epsilon} - I\frac{1-\delta-\epsilon}{2-\epsilon} - V\bigg) \\
&\qquad - (I+P)^2\frac{1-p}{2-\theta} + (A+V)\frac{1+\delta}{2}\bigg], \\
\dot{V} &= (1-p)V\bigg[A\frac{1+\delta}{2} - V\frac{1-\delta}{2} \\
&\qquad - I + (I+P)\big[A+V+p(I+P)\big]\frac{1+\delta}{2-\epsilon} - (1-p)(I+P)^2\frac{1}{2-\theta}\bigg], \\
\dot{A} &= (1-p)(I+V)\bigg[V\frac{1-\delta}{2} - A\frac{1+\delta}{2}\bigg].
\end{aligned}
\tag{2.1}
$$

Comparisons of the stochastic simulations and Eqs. (2.1) are provided in Fig. 2.

Equations (2.1) support multiple stable equilibrium states, including

$$
\begin{aligned}
S_I &:= (P,I,V,A) = \bigg(1 - \frac{1+\delta}{2-\epsilon} + \frac{1-p}{p(2-\theta)}, \frac{1+\delta}{2-\epsilon} - \frac{1-p}{p(2-\theta)}, 0, 0\bigg), \\
S_D &:= (P,I,V,A) = \bigg(0, 0, \frac{1+\delta}{2}, \frac{1-\delta}{2}\bigg), \quad \text{and} \\
S_U(P_0) &:= (P,I,V,A) = (P_0, 0, 0, 1-P_0).
\end{aligned}
$$

States S_D and $S_U(P_0)$ are equivalent to Dystopia and Utopia as described in [14]. However, S_I, a semi-utopian state in which all players cooperate with authorities but there is still a non-zero level of crime, is new to this version of the game. Furthermore, S_I appears from simulations to be the global attractor for the system when $I(0)>0$ and the connectivity parameter p satisfies the inequality

$$
p > \frac{2-\epsilon}{2-\epsilon+(2-\theta)(1+\delta)} =: p^*.
$$

When this is the case, all of the utopian equilibria S_U are unstable. For $p=p^*$, S_I intersects with $S_U(1)$ and loses stability in a transcritical bifurcation. Hence, at the connectivity level $p=p^*$, the equilibrium $S_U(1)$ gains stability. As $p<p^*$ is reduced towards zero, the I

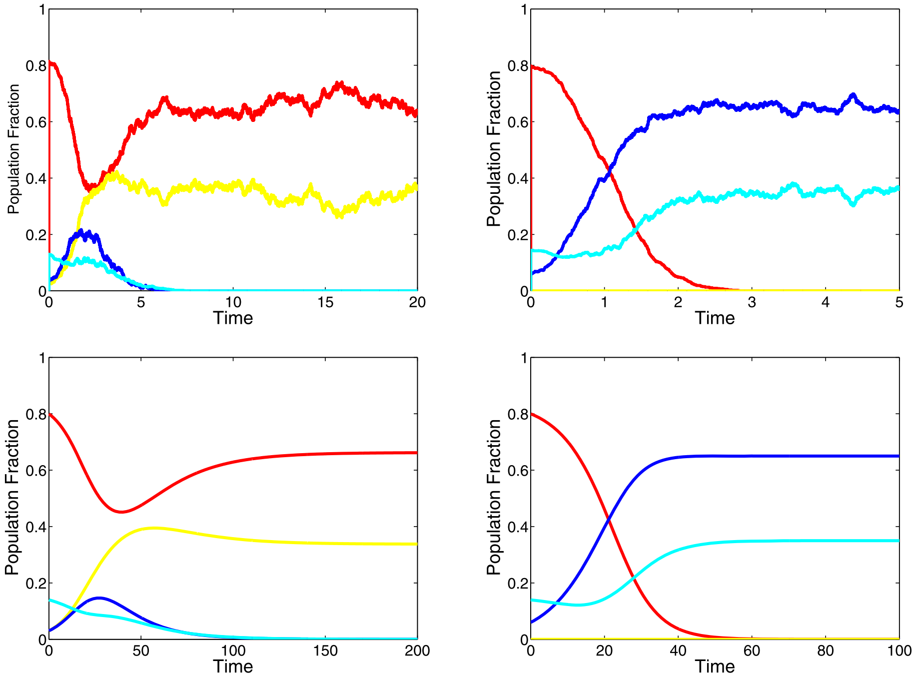

Fig. 2 Comparison of the stochastic (*top*) and continuous (*bottom*) models: P is *red*, I is *yellow*, A is *cyan*, and V is *blue*. The parameters are the same as in Fig. 1. The initial population fractions *on the left* are $(P, I, V, A) = (0.8, 0.03, 0.03, 0.14)$ and *on the right* are $(P, I, V, A) = (0.8, 0.0, 0.06, 0.14)$ (Color figure online)

component of S_I becomes negative and more of the states $S_U(P_0)$ gain stability in a series of transcritical bifurcations. However, the large attracting manifold of S_I persists and appears to guide the solutions to a limited subset of the stable solutions $S_U(P_0)$.

If $I(0) = 0$, then $I(t) = 0$ for all time and S_I is no longer an attractor. The long term dynamics then are dictated by the initial size of $P(0)$. $\dot{P}$ changes sign at the critical value

$$P_s := \frac{(1+\delta)(2-\theta)}{(1-p)[2-\epsilon+(1+\delta)(2-\theta)]}.$$

If $P(0) > P_s$, then the solution will progress towards one of the utopian equilibria $S_U(P_0)$, but if $P(0) < P_s$, then the solution will go towards Dystopia at S_D (see Fig. 3). There is also a saddle point of the system given by $(P, I, V, A) = (P_s, 0, V_s, A_s)$, where

$$A_s = \frac{(1-P_s)(1-\delta)}{2},$$
$$V_s = 1 - P_s - A_s,$$

though this equilibrium is not expected to be observed, and is not seen numerically.

The inclusion of LSVNs alters the dynamics of the game in two important ways. First, the threshold P_s for Utopia to develop in the absence of Informants has been scaled by the factor $1/(1-p)$. This vastly increases the basin of attraction for the Dystopia fixed point S_D for even moderate values of the connectivity. Second, Informants no longer universally drive the system towards Utopia. If $p > p^*$, any initial condition with a nonzero Informant

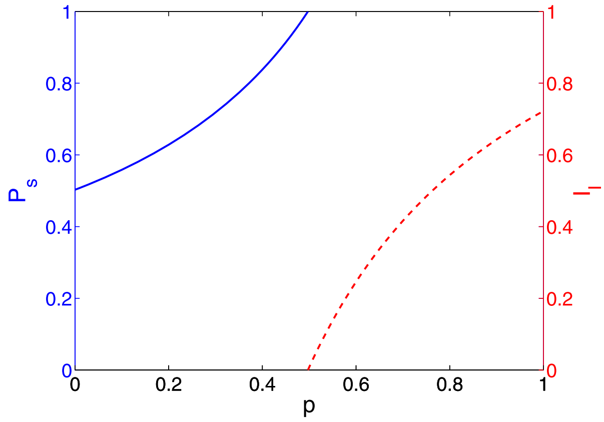

Fig. 3 The critical Paladin population P_s is plotted as a function of the LSVN connectivity p in *solid blue*. In the absence of Informants, the initial Paladin fraction of the population must lie above the *blue curve* in order for the system to evolve towards Utopia. The Informant population for the attracting state is plotted as a function of p in *dashed red*. Below p^* a Utopian state is the attractor, and above it S_I is the attractor. Both curves are for the same parameters as in Fig. 1

population will progress towards S_I, which is only a semi-utopian state, and has a stable population of criminals. Furthermore, under this condition and within the invariant plane $I = 0$, most of the phase space will be attracted to the S_D equilibrium because of the large value of p. Thus, the addition of sacred values has increased the likelihood of crime in the equilibrium state.

3 Local Victim Protection, Extended Criminal Protection

The model discussed in Sect. 2 is too simplistic to account for some of the behavior we are interested in studying. For the parameters from Figs. 1 and 2, the critical connectivity in the ER graph whereby a non-zero Informant population is stabilized, given by $p^* \approx 0.5$, is larger than could be reasonably expected from a real world network. This large p^* would imply that, in a population of 1000 people, everyone would need to share an ideology or kinship with at least 500 other people in order for a minority of criminals to remain in the society. This may be possible in the setting of terrorist networks, where the shared ideology provides protection, but clearly unrealistic for criminal gangs and organized crime. Also, there is no concept of a criminal coalition or gang within the LSVN formulation, outside of one's direct connections.

Yet, the LSVNs provide a framework by which one can define criminal coalitions (gangs). Consider a specific criminal player and his LSVN. If there is a second criminal in this LSVN, the two players are considered members of the same gang. Furthermore, if the second criminal is in the same gang as a third criminal, who is not in the LSVN of the first player, all three criminals are still considered to be in the same gang (see Fig. 4 for a schematic of this). This does not contradict our intuition about organized crime networks; the individual members in the network may only be personally connected to a few other members, yet the organization still exists as a larger construct. These sparse connections may provide protection against Informants taking down an entire organization, as individuals only have information about a small subset of the network [5]. However, this assumption may be less apt for gangs, since gang members often consider the entire gang as their family [17]. However, the dynamics could still organize themselves such that only connected clusters form gangs and everyone in the gang is in fact connected to most other gang members. To summarize, then, our concept of a gang as it relates to LSVNs is a group of criminals who are connected either directly or indirectly through *criminal* relationships.

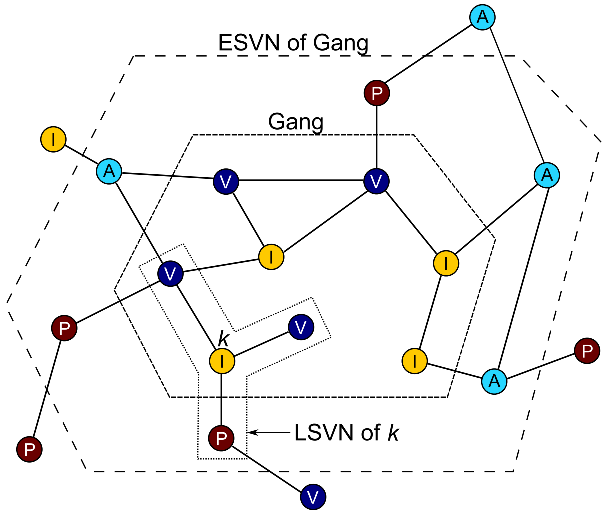

Fig. 4 A schematic of our definitions of gangs, LSVNs, and ESVNs, given the structure of the graph and the player types of each node. The nodes are labeled by player type: P for Paladin, A for Apathetic, I for Informant, and V for Villain

Being in a gang and being a friend of a gang member should affect how the game proceeds, both in how victims are selected and how trials occur. The defining structure of the gang has been prescribed above, and now the actual dynamics of the game with the gang structure needs to be defined. As in Sect. 2, friends of gang members should expect some reprieve from crime. In some settings, gangs may regulate crime against community property and community members in part to limit the additional police scrutiny that comes with crime [12]. However, friends also would be expected to protect the gang in legal matters, whether out of fear or loyalty. Remember that gang members are also siblings, sons, daughters, or grandchildren deserving of protection by their family members [12]. Furthermore, the gang members are likely to be better known within the community than are every single personal relationship of each individual gang member. For this reason, we assume friends of criminals enjoy the same protection as before (they will not be victimized by anyone in their LSVN, but still might be victimized by other members of the gang), but any member of the LSVN of any of the individual gang members will not witness against anyone in the gang. In other words, the protection against victimization is local, but the gang's protection from the community is more global; this is consistent with the observations in [6, 12]. The union of all the LSVNs of the gang members will be referred to as the extended sacred values network (ESVN) of the gang (see Fig. 4). The local nature of the victimization protection will enable gangs to merge as well as split. Our exploration of this model will rely solely on numerical simulation.

Within this version of the model, a dystopian fixed state with $\approx N(1+\delta)/2$ Villains and $\approx N(1-\delta)/2$ Apathetics is expected to exist. This is because, in these circumstances, the fact that criminal protection has been extended to the entire ESVN of a gang is irrelevant, since none of the players would witness anyway. Furthermore, if a single "super-gang" exists that contains all criminals, and the ESVN of this gang extends to roughly all N players, then we might expect to observe states in which the total number of criminals is of size $\approx N(1+\delta)/2$—but may consist of both Villains and Informants—and the total number of non-criminals is of size $\approx N(1-\delta)/2$—but may consist of both Apathetics and Paladins. This is because, if the ESVN of the super-gang is of size N, all trials are unsuccessful. Hence, Paladins and Informants are essentially the same as Apathetics and Villains, respectively, except for the fact that they fare somewhat worse upon being victimized (due to the

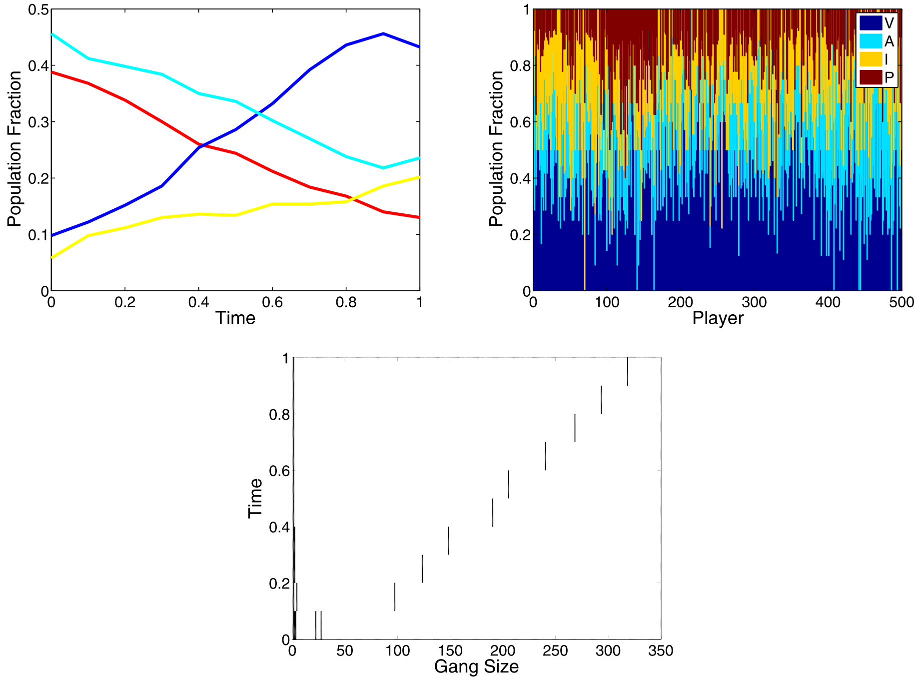

Fig. 5 Example realization of the ESVN model, run until the moment a super-gang first forms, using $N = 500$, $p = 0.016$, and all other parameters the same as in Fig. 1. P is *red*, I is *yellow*, A is *cyan*, and V is *blue*. On the *top right* is a plot of the LSVN makeup of each player, with indices ordered based on current strategy: approximately the first 100 players are Informants, the next 60 Paladins, then 220 Villains, and lastly 120 Apathetics. The *bottom figure* provides a simplified histogram of gang sizes over time. A *black mark* denotes the existence of at least one gang with that size at that time point (Color figure online)

ϵ retaliation loss they suffer). In order to potentially observe such states, our simulations are generally executed with a local connectivity p that is sufficiently large so that a giant component of size at least $\approx N(1+\delta)/2$ is expected to exist in the ER graph (often they are fully connected); thus, every criminal could potentially be connected to a single super-gang.

The ESVN model arranges player types heterogeneously, unlike the local model from Sect. 2. This can be seen by looking at the makeups of the LSVNs of each player, sorted by player type, as shown in Figs. 5 and 6. In Fig. 5, a super-gang containing $\approx N(1+\delta)/2$ nodes forms, but is composed of both Informants and Villains. Interestingly, the LSVNs of Paladins in this case are more likely to contain Paladins than are the LSVNs of the other types. Similarly, the LSVNs of Villains in this case are significantly more likely to contain other Villains than are the LSVNs of, say, Apathetics. In Fig. 6, we observe an even stronger tendency for Villains to be directly connected to other Villains. Here, Informants are absent, and the system enters Dystopia. In this simulation, however, the largest gang does not contain all of the criminals, due to the much lower p value used. Because of the higher-order correlation between players that leads to clustering, the ESVN model is unlikely to succumb to a simple ODE analysis, unlike the LSVN model.

Despite the formation of a super-gang, if the gang contains a sufficient number of Informants, it may still fracture. See Fig. 7 for a sample realization of this phenomenon. Though Informants may thus lead to the emergence of Utopia as the final state, the system can

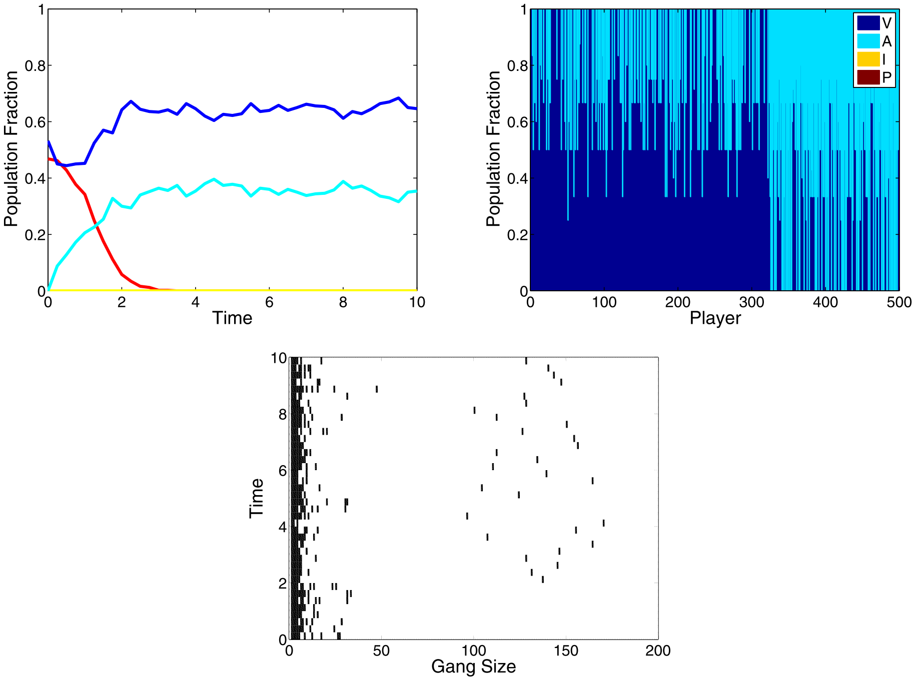

Fig. 6 A sample realization for the ESVN model with $N = 500$, $p = 0.0039$, and initial population fractions $(P, I, V, A) = (0.5, 0.0, 0.5, 0.0)$. All other parameters are the same as in Fig. 1. As with other figures, P is *red*, I is *yellow*, A is *cyan*, and V is *blue*. In the plot of LSVN makeup for each player, indices are sorted by player type, with roughly the first 320 players being Villains and the last 180 players being Apathetics. The *bottom figure* provides a simplified histogram of gang sizes over time. A *black mark* denotes the existence of at least one gang with that size at that time point (Color figure online)

maintain a super-gang formation for very long periods of time. Figure 8 illustrates this phenomenon. Here, the system is initialized with $P = 0.5$, a varying number of Informants, and all other citizens initially Villains. We then measure the fraction of realizations that end in Utopia within a specified time window. For a significant initial Informant population, most realizations eventually end in Utopia, but the time to reach that state can be quite long.

The formation of gangs in this model can be reduced to a question in random graph theory; hence, an equivalent problem on the distribution of connected components in ER random graphs could be studied. Consider an ER graph with connection probability p and number of vertices n, which develops in time. The factor n in this model would be given by the total criminal population $I + V$. At each time step, either a vertex is deleted, and the network size is decreased to $n - 1$, with this deletion probability being related to the size of the connected component that the vertex is contained in (and possibly the relative proportions of I and V), or an entirely new vertex is created ($n \to n+1$), which is connected to each other vertex with probability p. This problem appears to be related to the Barabási-Albert model of network formation via preferential attachment [4]. It would be interesting to see the distribution of connected component size in this model, and its development in time.

The formation of gangs can be examined from a different viewpoint as well. Specifically, we note the largest gang size, as well as the total number of criminals N_c^i, for many different realizations i of the game at a fixed ending time (chosen to be $t = 10$ in this case), using the same underlying ER graph and initial population composition each time, but with different

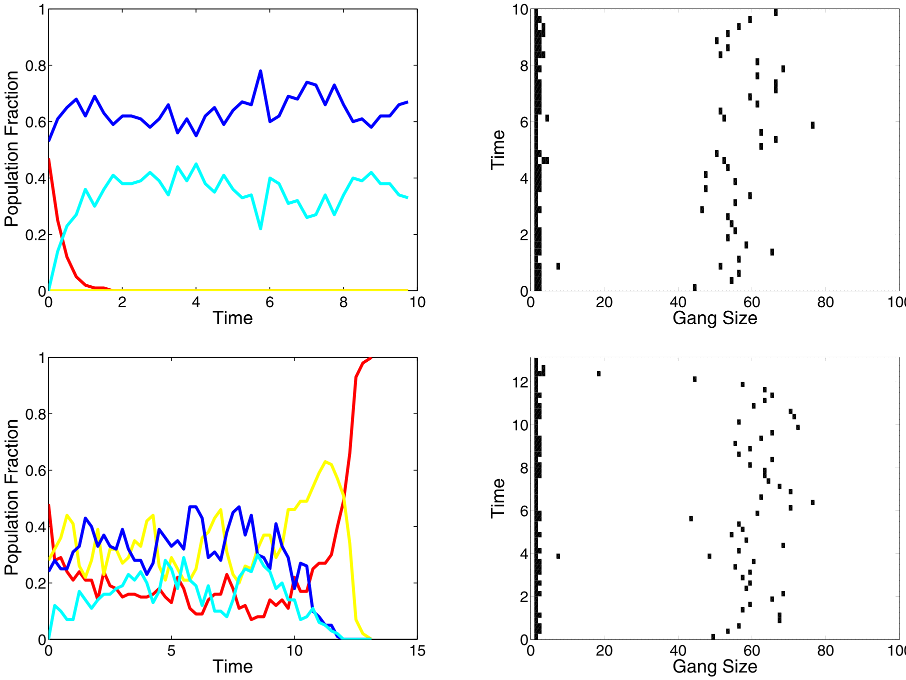

Fig. 7 A comparison between super-gang formation with and without Informants. The parameters are $N = 100$, $p = 0.04$, and initial population fractions of $(P, I, V, A) = (0.5, 0.0, 0.5, 0.0)$ *on top* and $(P, I, V, A) = (0.5, 0.25, 0.25, 0.0)$ *on bottom*. All other parameters are the same as in Fig. 1. P is *red*, I is *yellow*, A is *cyan*, and V is *blue* (Color figure online)

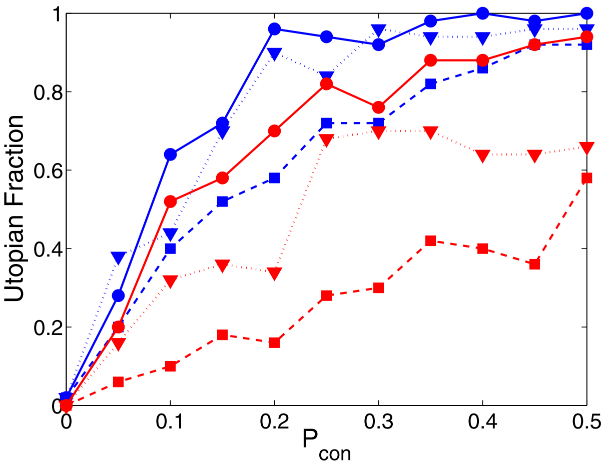

Fig. 8 The fraction of 50 realizations of the ESVN model that end in Utopia within the timeframes $T = 250$ (*circles* and *solid lines*), $T = 30$ (*triangles* and *dotted lines*), and $T = 10$ (*squares* and *dashed lines*) for p values $p = 0.04$ (*blue*) and $p = 0.045$ (*red*). Initial conditions are $(P, I, V, A) = (0.5, I_0, 0.5 - I_0, 0)$ and $N = 100$; all other parameters are as in Fig. 1 (Color figure online)

initial placements of the players within the graph in each realization. We then go back and randomly place N_c^i criminals on the graph 5,000 times for each realization i, and determine what the largest connected component is in these cases; this is equivalent to creating an ER graph of size N_c^i with the same p as used in the simulation. This allows us to compare

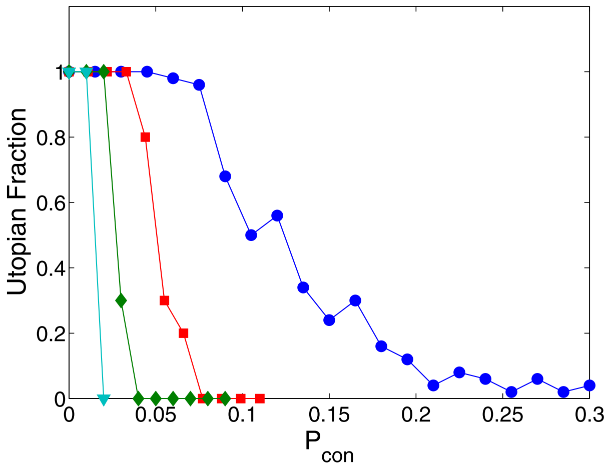

Fig. 9 The fraction of simulations ending in utopia for the ESVN model as a function of network size N and connectivity p, with initial population fractions $(P, I, V, A) = (0.8, 0.0, 0.1, 0.1)$. *Blue circles* indicate $N = 100$ (50 realizations), *red squares* indicate $N = 300$ (20 realizations), *green diamonds* indicate $N = 500$ (20 realizations), and *cyan triangles* indicate $N = 1000$ (10 realizations). All other parameters are the same as in Fig. 1. *Lines* are to guide the eye. The critical connectivity for sustained criminal activity decreases with N, but is above the threshold to expect a giant component

the observed largest gang size in each realization to the median randomly generated largest gang size for that number of criminals N_c^i. The result is that, of the 887 such realizations that we ran, 456 displayed a largest gang size greater than the randomly generated median for that N_c^i, while 423 were smaller and 8 were equal. Hence, the largest gang sizes are not statistically different than those expected by chance under this sort of ensemble averaging.

By extending the protection against witnessing that criminals feel to the entire ESVN, much lower connectivities p are needed to sustain crime than in Sect. 2 (see Fig. 3), in the absence of Informants. This is because, when a criminal is part of a gang, he is effectively connected to a very large portion of the network even though he has relatively few close ties. In Fig. 9, the likelihood of progressing to Utopia from a fixed small initial Villain population of $V = 0.1$ is studied as a function of network size N and connectivity p. The utopian fraction is calculated as the fraction of realizations attaining a utopian state before a fixed ending time. The ending time was chosen sufficiently large so that the level V reached a value of approximately $(1 + \delta)/2$ in most realizations that did not reach Utopia. The necessary connectivity to sustain crime decreases with increasing N, but in each case is larger than the connectivity required to expect a giant component. This is not surprising, given the Paladin-dominated initial conditions used here. For moderately sized graphs, the connectivities are perfectly reasonable for human behavior. For example, at $N = 500$, each player needs to be connected to roughly 20 or more other people in order for a dystopian state and a gang to stabilize.

4 Fully Extended Protection

In certain settings, one may expect shared ideology to be a protective force against both witnessing and criminal attacks, thus differing from the purely local victimization protection discussed in Sect. 3. In this section, we will therefore examine the situation where the pro-

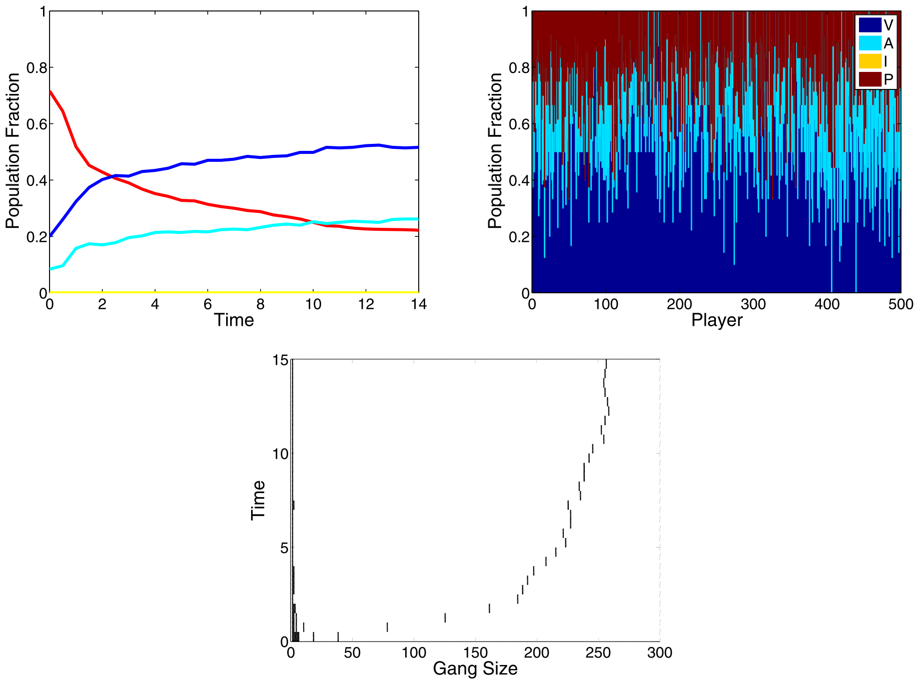

Fig. 10 The protected ESVN model with an underlying ER graph. The parameters are $N = 500$, $p = 0.016$ and initial population $(P, I, V, A) = (0.7, 0.0, 0.2, 0.1)$. All other parameters are the same as in Fig. 1. P is *red*, I is *yellow*, A is *cyan*, and V is *blue* (Color figure online)

tection against victimization is extended to the entire ESVN of a gang, as protection against witnessing was so extended in Sect. 3; we will term this the protected ESVN model.

Extending victimization protection to the entire ESVN of a gang has a very large impact on the model. For example, if exactly two gangs are present, they cannot merge directly, in contrast to the model in Sect. 3. This is because, directly prior to the two gangs merging, there must exist at least one non-criminal "bridge node" that is directly attached to both gangs. However, this node would therefore belong to the ESVN of both gangs, and so would never be victimized (remember that we assume exactly two gangs). Hence, that bridge node will never change strategies; specifically, it will never become a criminal type that would then merge the two gangs into one. Because of this, we may expect to find dystopian final states of the system that exhibit two large gangs, rather than just the single large gang typically observed in Sect. 3. On the other hand, the number of gangs may still decrease if a gang is simply wiped out, and its members converted into non-criminal types.

The protected ESVN model will also allow final states that are neither purely utopian nor purely dystopian. This is because the ESVN of a gang may contain, for example, Paladins, but that will retain their strategy due to the protection afforded them by virtue of their ESVN membership. Figure 10 displays a typical simulation of the protected ESVN model. Here, one super-gang forms, and the paladin population is quickly reduced. However, the level of V is slightly lower than $(1 + \delta)/2$, and P remains nonzero due to the extended protection from victimization.

The main advantage of using an ER graph in Sect. 2 was to allow us to easily write down a system of ODEs to model the dynamics. The extended model, however, is not homogeneous

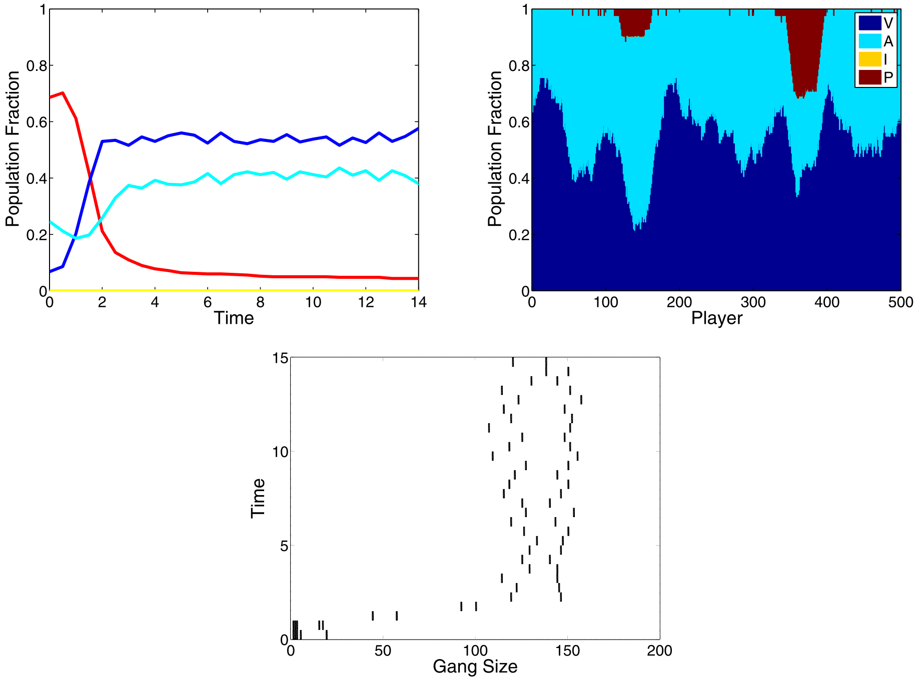

Fig. 11 The protected ESVN model on a small world network. Here $N = 500$, $k_{\mathrm{sw}} = 20$, $p_{\mathrm{sw}} = 0.4$, and initial population fractions are $(P, I, V, A) = (0.7, 0, 0.07, 0.23)$. All other parameters are the same as in Fig. 1. Two stable gangs form, and the Paladin population decays slowly (Color figure online)

enough for a similar analysis to be useful. In addition, for an ER graph, producing two distinct gangs in the final state, as alluded to above, is difficult. This is because, in order for the system to progress towards Dystopia, a critical number of criminals must be reached. Once this critical number is reached, however, it is nearly impossible for all of the gangs to be disconnected, unless the p value is too low to allow for a super-gang to form in the first place. Hence, if Dystopia is reached with an ER graph of high enough p, it almost certainly contains only one super-gang.

It is therefore worthwhile to consider other random graph models for the underlying sacred values networks, such as a typical small world network (e.g., [18]). For such a network, let the clustering parameter and rewiring parameter be referred to as k_{sw} and p_{sw}, respectively. Using a small world graph, several of the local hubs can be completely populated by criminals without worrying that each hub is directly connected to each other. If these hubs are large enough, crime should persist, leading to multiple stable gangs. Figures 11 and 12 are typical realizations of the protected ESVN game on a small world network both without and with Informants, respectively. The initial criminal populations are distributed randomly in both cases, and initially many gangs are present. All but two of these gangs quickly die out, and these two competing gangs persist for long times. In Fig. 11, two large clusters of Paladins are also seen to persist. The Paladin populations are protected by the individual gangs in whose ESVN they reside, but are expected to slowly die out as time goes on and they are victimized by the other gang. In the case with Informants (Fig. 12), the crime level eventually drops below a sustainable threshold and the gangs fractionate as in the previous model. The system then progresses to Utopia.

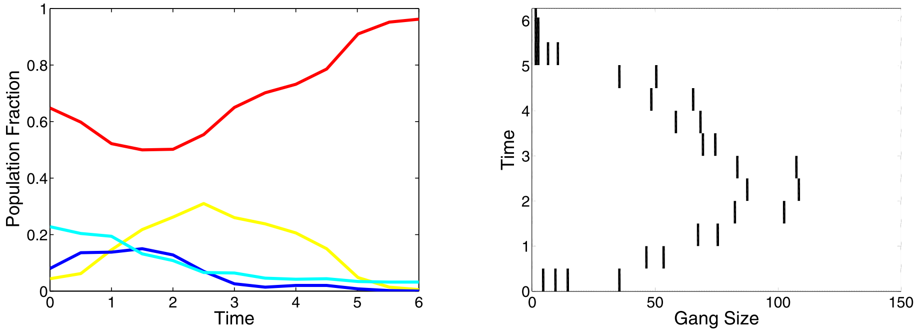

Fig. 12 The protected ESVN model on a small world network. Here $N = 500$, $k_{sw} = 20$, $p_{sw} = 0.1$ and initial population fractions are $(P, I, V, A) = (0.7, 0.05, 0.07, 0.18)$. All other parameters are the same as in Fig. 1. Two gangs form, but the Informants eventually drive the entire system to utopia (Color figure online)

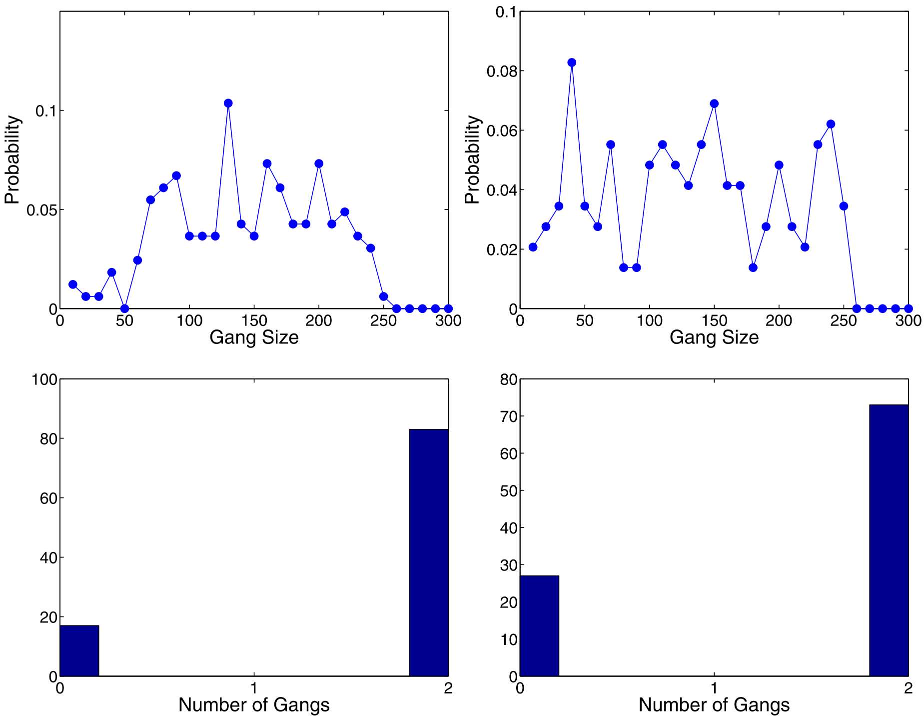

Fig. 13 Histograms of gang sizes and number of gangs for the protected ESVN model on a small world network with $N = 500$, $k_{sw} = 20$, and $p_{sw} = 0.1$ (*left*) or $p_{sw} = 0.4$ (*right*). The initial population is given by $(P, I, V, A) = (0.7, 0.0, 0.1, 0.2)$ *on the left* and $(P, I, V, A) = (0.7, 0.0, 0.07, 0.23)$ *on the right*. All other parameters are the same as in Fig. 1. Two gangs tend to form and stabilize in a majority of runs

On a small world network with random initial data, two gangs appears to be the stable distribution. This was tested for $N = 500$ and $k_{sw} = 20$ by seeding a small initial population of criminals and examining the distribution of numbers of gangs and gang size after long times, for two values of the parameter p_{sw}; this is displayed in Fig. 13. In both cases, two gangs is the dominant outcome, with the other outcome being Utopia. When two gangs form, they are generally of unequal size, and there appears to be structure in the histogram of gang sizes. This histogram was binned with width 10, which is half of the expected cluster size for the small world graph. Gangs appear to be formed by completely populating individual clusters, which explains the strong oscillation between consecutive bins.

5 Discussion

Personal relationships or protected social statuses can allow crime to flourish in otherwise peaceful communities. We refer to these personal relationships or protected social statuses as "sacred values", which are considered by their holders to be inviolable or non-negotiable. We explored this phenomena in the context of an evolutionary, adversarial game on an inhomogeneous network where certain network connections encode sacred values. The inclusion of sacred values into a stochastic game allowed crime to persist and flourish from extremely low crime states. Sacred values networks lead to a natural definition of a gang, and the development and stabilization of such structures was also seen. In many cases, the inclusion of an Informant type could help destroy a gang's structure, but the Informant population itself could also become a stabilized criminal element.

In the simplest model (LSVN), we were able to fully characterize the system dynamics through a continuum limit of the stochastic game. For the extended models, such an analysis was not possible due to the large inhomogeneities in the structure of the local networks. A full study of dynamic random graphs in which we create or delete a vertex at each time step would be useful to understand our model. The likelihood of deletion would necessarily depend on the distribution of connected components in ER random graphs well below the connectivity threshold.

Many refinements to the model could be developed. The simplest improvement would be to use more realistic relationship graphs for the underlying sacred values network. We concentrate on ER graphs because their properties allow certain simple analyses, but they are in general not a good model for social networks. The small world graph is an improvement, and interesting behavior is seen on small world networks that is not seen on ER graphs, but they are still not an ideal model for social networks. Another interesting direction would involve pairing a dynamic sacred values network with the altered gang dynamics. The changing network could be defined from a weighting function on vertices with feedback from the state of each player.

Acknowledgements This work was supported by ARO MURI grant W911NF-11-1-0332, NSF-FRG grant DMS-0968309, and AFOSR-MURI FA9550-10-1-0569.

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